# PART 1: Is the Collatz Conjecture Indeed Indecipherable?

## Introduction

Scheme 1:

| | |
|---|---|
| /2 | The Collatz (1937) conjecture formulation: Start with any positive integer. |
| S → E < — > E | Calculation rules (CC computing): 1) the number is even: divide by two. |
| /2 ↓↑ x3+1 | 2) The number is odd: triple it and add one (Scheme 1). A sequence by performing |
| S → O | computations repeatedly will eventually converge to 1.The conjecture is false, |
| ↓ x3+1 | if there is some starting number which would enter to a repeating cycle that |
| S → EB → 1 | does not contain 1 or increase without bound. The final decision is missing, and |
| S= start, E=even, | the Collatz conjecture is one of the most famous unsolved problems in mathematics. |
| O= odd, B= binary | The effort goes on. There is even a reward for solving the problem (a 120M Japanese Yen). The candidate is assumed to publish the solution in a reputable Journal. |

## Methods

The reasoning that breaks the problem down into pieces and explains a chain of thoughts step by step. This process is based on known and evident facts, elementary mathematics (if any), graphical representations (TABs) with verbal descriptions, examples, and explanatory remarks.

## A. Solutions

**A S1.**

**Fact 1**: Every non-negative integer is a summation series of multiples of powers of the base (Z) of a number positional system in which the distinction between two consecutive terms is a constant value (d) – arithmetic (A) series, or multiple (q) – geometric (G) series.

Example 1:

a. 203 (Z= 2, q= 2) 1x 2^7 + 1x 2^6 + 0x 2^5 + 0x 2^4 + 1x 2^3 + 1x 2^1 + 1x 2^0

b. 203 (Z= 4, d= 4) 3x 4^3 + 0x 4^2 + 2x 4^1 + 3x 4^0

**A S2.**

**Fact 2**: Four arithmetic sequences (ASQs a-d) contain the entire set of positive integers:

TAB 1

| | | | |
|---|---|---|---|
| a | ASQ | 0, d= 4 | 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64 |
| b | ASQ | 1, d= 4 | 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65 |
| c | ASQ | 2, d= 4 | 2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42. 46, 50, 54, 58, 62, 66 |
| d | ASQ | 3, d= 4 | 3, 7, 11, 15, 19, 23, 27, 31, 35, 39, 43, 47, 51, 55, 59, 63, 67 |
| | Ordering numbers | | 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16 |
| e | GSQ | 1, q= 2 | 1, 2, 4, 8, 16, 32, 64,128, 256 Binary row |
| f | GSQ | 1, q= x4 + 1 | 1 5 21 85 ↑ x3 + 1 |

**Fact 3**: All values of the CC computing [(2n+1) x 3] +1 (TAB 2) are part of two ASQ - a, c.

The determination of a sequence to which a number belongs: If p, r are non-negative integers and p>r, then the remainder at p: r is from the interval {0, r-1}.

Example 2:

**a**. 173 173:4= 43 + remainder 1 (calculator 43.25) → the number from the b ASQ, because the remainder 0.25=1:4.

**b**. 871 (178 CC steps – Wikipedia) 871:4= 217.75 is the number from the d ASQ, because the remainder 0.75= 3:4.

**Consideration 1**: The values of the ASQ a-d tend toward ∞. This goal is unattainable, because there is no such large odd number that cannot be followed by an even number and its single or multiple divisibility /2 – shift toward 1 (TAB 2).

TAB 2

| | | |
|---|---|---|
| Divisibility of values x3 + 1 | | 1 5 1 11 7 17 5 23 13 29 1 |
| ASQ | 4, d= 6 | **4**, 10, **16**, 22, **28**, 34, **40**, 46, **52**, 58, **64** ↑ /2 |
| ASQ | 1, d= 2 | 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21 ↑ (2n+1) x 3] +1 |

Remark 1: Bold even numbers are divisible /2 two or more times

**Consideration 2**: A certain number of steps (d= 4) leads in the rows a-d (TAB 1) to the initial values of each SQ (0, 1, 2, and 3).

**Consideration 3**: This movement does not generate any cycles.

**A S3**.
**Fact 4**: The final steps – CC computing until the resulting value is 1.

TAB 3

| | |
|---|---|
| 0= 4 | /2 /2= 1 |
| 1= | x3+ 1 = 4; 4 /2= 2; 2/2= 1 |
| 2= | /2= 1 |
| 3= | x3+ 1= 10; 10 /2= 5; 5 x3 + 1= 16; 16/2= 8; 8/2= 4; 4/2= 2; 2/2= 1 |

**B**. The results published so far indicate <u>an intrinsic tendency to shift toward 1;</u> the reason can be merely the CC operator properties.
Remark 2: Some parts of B are more empiric-relational.

**Fact 5**: Binary GSQ (TAB 1, e) is the only one that, using the operator /2, leads to 1.

**Fact 6**: The entry gateway to this SQ is the operator x3 + 1 (TAB 1, f)

TAB 4

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | 16 | | | | 64 | 2^n Binary |
| | | | ↑ x3+1 | | | | ↑ x3+1 | |
| 1 > | x4+1 | > | 5 | > | x4+1 | > | 21 | TAB 1, GSQ f |
| | | | ↓ x2 | | | | ↓ x2 | |

**B 1.**
TAB 5
Number of "entry gateways" in the interval {5, 1024}:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | Binary 2^n | | | |
| → /2 | 682 | | | | | | 341 | x3 +1 | 1024 | | | 512 |
| → /2 | 680 | 340 | 170 | | | | 85 | → | 256 | | | 128 |
| → /2 | 672 | 236 | 168 | 84 | 42 | | 21 | → | 64 | | | 32 |
| 640 | 320 | 160 | 80 | 40 | 20 | 10 | 5 | → | 16 8 | 4 | 2 | 1 |
| ↑ | x 3+1 | ↑ | | ↑ | | | | | | | | |
| <u>213</u> | | <u>53</u> | x 2 | <u>13</u> | | | | | | | | |
| n= 2 | | n= 4 | ↓ | n= 6 | | | | | | | | |

**Consideration 4**: The probability of entry increases by narrowing the interval and moving it to lower values: {5, 1024} 48:1024= 0.05; {5, 16} 3:16= 0.2.

Remark 3: With an unlimited number of experiments, if an event has non zero probability of occurring, then the probability that this event happen at last ones is equal 1.

**B 2.**
In the next part of the presented text, <u>the code of CC operators</u> is used: x 3+ 1 = 1 /2 = 0
Example 3:

| **a** | | | | **b** | | | |
|---|---|---|---|---|---|---|---|
| **1** | **0** | **1** | **0** | **1** | **0** | **0** | **1** |
| 27 | 82 | 41 | 124 | 33 | 100 | 50 | 25 |

Remark 4: The codes are not the actual values; they may be the same for different value intervals (TAB 7).

**Fact 7**: The Collatz problem formulation allows operator sequences 00, 01, and 10, but excludes 11 (an impossible event); this limits the number of combinatory possibilities.

**Fact 8**: The allowed operator sequences in the groups of four items:
TAB 6

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **2/4** | **a** | 1 | 0 | 1 | 0 | **1/4** | 0 | 0 | 0 | 1 |
| | **b** | 0 | 1 | 0 | 1 | | 0 | 0 | 1 | 0 |
| | **c** | 1 | 0 | 0 | 1 | | 0 | 1 | 0 | 0 |
| **4/4** | | 0 | 0 | 0 | 0 | | 1 | 0 | 0 | 0 |

Remark 5: The possibilities of placing two fours after each other: 5 fours are ending in 0; all eight fours can pursue it: 5x8 = 40 options. 3 fours end in 1; five fours starting with 0 can pursue it: 3x5 = 15. The total: 55 eight-member groups.
Remark 6: The arrangement of CC operator codes in 8-member groups allows us to consider these groups as 8-bit bytes and to use lossless binary data compression techniques (55= 6 bits).

**B 3.**
**Consideration 5**: The ratio of values (last–first) in each four can define the shift to 1, or to ∞;

B 2, Example 3a= 124:27= 4.6 (shift toward ∞); B 2, Example 3b= 25:33= 0.76 (shift toward 1).
Example 4:
TAB 7

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **2/4 a** | **1** | **0** | **1** | **0** | ∞ ← | **2/4 b** | **0** | **1** | **0** | **1** | → 1 |
| | 27 | 82 | 41 | 124 | 4.6 | | 22 | 11 | 34 | 17 | 0.77 |
| | 103 | 310 | 155 | 466 | 4.5 | | 62 | 31 | 94 | 47 | 0.75 |
| | 719 | 2158 | 1079 | 3238 | 4.5 | | 526 | 263 | 790 | 395 | 0.75 |
| | 1367 | 4102 | 2051 | 6154 | 4.5 | | 2078 | 1039 | 3118 | 1559 | 0.75 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **2/4 c** | **1** | **0** | **0** | **1** | → 1 | **4/4** | **0** | **0** | **0** | **0** | → 1 |
| | 17 | 52 | 26 | 13 | 0.76 | | 16 | 8 | 4 | 2 | 0.125 |
| | 121 | 364 | 182 | 91 | 0.75 | | 48 | 24 | 12 | 6 | 0.125 |
| | 433 | 1300 | 650 | 325 | 0.75 | | 80 | 40 | 20 | 10 | 0.125 |
| | 20737 | 62212 | 31106 | 15553 | 0.75 | | | | | | |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | → 1 |
| **1/4** | 0 | 0 | 0 | **1** | 8 | 4 | 2 | **1** | 0.125 |
| | 0 | 0 | **1** | 0 | 12 | 6 | **3** | 10 | 0.8 |
| | 0 | **1** | 0 | 0 | 10 | **5** | 16 | 8 | 0.8 |
| | **1** | 0 | 0 | 0 | **21** | 64 | 32 | 16 | 0.8 |

TAB 8
Example 5:
**27** 111 steps– the display of 27 CC computing is in the PART 2:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1010 | 0101 | 0101 | 0100 | 1001 | 0100 | 1010 | 1001 | 0101 | 0100 |
| 1000 | 1010 | 1001 | 0100 | 1010 | 1010 | 1010 | 0010 | 1010 | 1000 |
| 0100 | 1001 | 0000 | 1000 | 1010 | 1000 | 0010 | 001 | | |

Sequences of the operators in Collatz computing starting at 27:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2/4 | a 1010 | 7 | b 0101 | 3 | c 1001 | 4 | | |
| 1/4 | 0001 | 1 | 0010 | 2 | 0100 | 5 | 1000 | 4 |
| 4/4 | 0000 | 1 | | | | | | |

Remark 7: The process continues until it ends at 1, even though that the values of the CC computing can be high in some computation phases (9.232 for start number 27).

**Consideration 6**: Only sequence 1010 of the CC computing makes a shift toward ∞ (see Consideration 1).

## Conclusions

The entire set of non-negative integers contained by four arithmetic sequences always reaches 1, no matter which integer starts the sequence. The Collatz conjecture is a decidable problem in the interval {1, infinity}.

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PART 2: The End of the "Syren Song" on the Single Page

Remark 1: The Collatz conjecture formulation and computing (CC) see PART 1- **Introduction**.

Remark 2. The CC formulation does not specify a number system to work in.

Remark 3: The "millennium" problem. Hundreds of articles in the renowned journals (Wikipedia).

27 CC 111 steps
27, 82, 41, 124, 62, 31, 94, 47, 142, 71, 214, 107, 322, 161, 484, 242, 121, 364, 182, 91, 274, 137, 412, 206, 103, 310, 155, 466, 233, 700, 350, 175, 526, 263, 790, 395, 1186, 593, 1780, 890, 445, 1336, 668, 334, 167, 502, 251, 754, 377, 1132, 566, 283, 850, 425, 1276, 638, 319, 958, 479, 1438, 719, 2158, 1079, 3238, 1619, 4858, 2429, 7288, 3644, 1822, 911, 2734, 1367, 4102, 2051, 6154, 3077, 9232, 4616, 2308, 1154, 577, 1732, 866, 433, 1300, 650, 325, 976, 488, 244, 122, 61, 184, 92, 46, 23, 70, 35, 106, 53, 160, 80, 40, 20, 10, 5, 16, 8, 4, 2, 1

**Fact 1**: 6 iterations can be enough

## Solutions

The only way to prove the Collatz conjecture is to show directly that every positive integer goes to 1. In terms of CC, two SQs lead to 1 with the only calculation /2: a) the binary positional system (Leibniz 1703). b) The 4^n row.

| 4^0 | 4^1 | 4^2 | 4^3 | 4^4 | 4^5 | 4^6 | 4^7 | row 4^n {1, infinity} |
|---|---|---|---|---|---|---|---|---|
| 0 | ***0*** | 0 | 0 | 0 | 0 | 0 | 0 | 0; 1; 2= coefficients |
| 1 | 1 | ***1*** | ***1*** | 1 | 1 | 1 | 1 | |
| ***2*** | 2 | 2 | 2 | 2 | 2 | *2* | 2 | 27 x3+1 = 82 = (2, 0, 1, 1) |

| | | | | | | |
|---|---|---|---|---|---|---|
| 82= 2x 4^0 + | 0x 4^1 + | 1x 4^2 | + | 1x 4^3 | | Procedure description: a) Write an even decimal number |
| 82= 2 + | 0 + | 16 | + | 64 | | on 4^n base. b) Divide by two the maximal element |
| /2 | | /2 | | /2 | | (64 in this illustration). c) Repeat /2 until the result |
| 1 | | /2 | | /2 | | corresponds to the value of the next lower position with |
| | | /2 | | 16 | | non-zero occurrence. d) The process continues to the |
| | | 2 | | | | last division 2/2= 1. |

**Fact 2**: 27x 3+1= 82 → 1 = 7 steps, 6 iterations /2.

Remark 4: There are no cycles.

## Conclusions

The results prove that using the 4^n row, the Collatz conjecture is a decidable problem in the interval {1, infinity}.

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